Use Pascal's Triangle to expand and simplify $(n^3 - 2r^4)^5$. **SCORE: _____ / 5 PTS**

$$(n^3)^5 + 5(n^3)^4(-2r^4) + 10(n^3)^3(-2r^4)^2 + 10(n^3)^2(-2r^4)^3 + 5(n^3)(-2r^4)^4 + (-2r^4)^5$$

$$= \underbrace{n^{15}}_{\frac{1}{2}} \underbrace{-10n^{12}r^4}_{1} \underbrace{+40n^9r^8}_{1} \underbrace{-80n^6r^{12}}_{1} \underbrace{+80n^3r^{16}}_{1} \underbrace{-32r^{20}}_{\frac{1}{2}}$$

Using the formulae for the sums of the powers of integers in your textbook, find the sum $\sum_{n=1}^{240}(2-5n+3n^2)$. **SCORE: _____ / 4 PTS**

Your final answer may **NOT** use ... **NOR** $\sum$. It may use $+, -, \times, \div$. (It does **NOT** need to be simplified into a single number.)

$$\sum_{n=1}^{240} 2 - 5\sum_{n=1}^{240} n + 3\sum_{n=1}^{240} n^2$$

$$= \underbrace{240 \cdot 2}_{\frac{1}{2}} \underbrace{- 5 \cdot \frac{240(241)}{2}}_{1\frac{1}{2}} \underbrace{+ 3 \cdot \frac{240(241)(481)}{6}}_{2}$$

FJ started a 30 day treatment program which involved daily injections of a medication. The first day's injection **SCORE: _____ / 5 PTS**
was 12 mg, and each subsequent day's injection was 5% less than the previous day's injection. Find the total amount of the injections.
Your final answer may **NOT** use ... **NOR** $\sum$. It may use $+$, $-$, $\times$, $\div$ and powers. (It does **NOT** need to be simplified into a single number.)

a_n = AMOUNT OF INJECTION ON n^{TH} DAY

$a_1 = 12$

$a_2 = 12(1-0.05) = 12(0.95)$

$a_3 = 12(0.95)^2$

$\vdots$

$a_n = 12(0.95)^{n-1}$

1 $\quad$ $2\frac{1}{2}$

$$\text{TOTAL} = \frac{12(1-0.95^{30})}{1-0.95}$$

$1\frac{1}{2}$

Find the rational number representation of the repeating decimal $0.4\overline{27}$ using the method discussed in lecture. **SCORE: _____ / 6 PTS**
NOTE: Only the 27 is repeated.

$0.4 + 0.027 + 0.00027 + 0.0000027 + \dots$ (1)

$= \frac{4}{10} + \frac{0.027}{1 - \frac{1}{100}}$

$= \frac{4}{10} + \boxed{\frac{\frac{27}{1000}}{\frac{99}{100}}}$ (2½)

$= \frac{4}{10} + \frac{\overset{3}{27}}{1000} \cdot \frac{100}{\underset{11}{99}}$

$= \boxed{\frac{4}{10}} + \boxed{\frac{3}{110}}$ (½) (1)

$= \boxed{\frac{47}{110}}$ (1)

Use mathematical induction to prove that is 3 a factor of $n^3 + 6n^2 + 8n$ for all positive integers n. **SCORE: _____ / 10 PTS**

BASIS STEP: 3 IS A FACTOR OF $1^3+6(1)^2+8(1)=15$ ①

MUST HAVE WORD "INTEGER" ①

INDUCTIVE STEP: ASSUME 3 IS A FACTOR OF k^3+6k^2+8k ①

FOR SOME PARTICULAR BUT ARBITRARY INTEGER ① $k \geq 1$

[PROVE 3 IS A FACTOR OF $(k+1)^3+6(k+1)^2+8(k+1)$]

$(k+1)^3+6(k+1)^2+8(k+1)$ ①

$= k^3+3k^2+3k+1$
$\quad +6k^2+12k+6$
$\quad +8k+8$

$=(k^3+6k^2+8k)+(3k^2+15k+15)$

$=(k^3+6k^2+8k)+3(k^2+5k+5)$ ②

SINCE 3 IS A FACTOR OF BOTH (k^3+6k^2+8k) AND $3(k^2+5k+5)$ ①

THEREFORE 3 IS A FACTOR OF $(k+1)^3+6(k+1)^2+8(k+1)$ ①

SO, 3 IS A FACTOR OF n^3+6n^2+8n $\frac{1}{2}$

FOR ALL INTEGERS $n \geq 1$ $\frac{1}{2}$